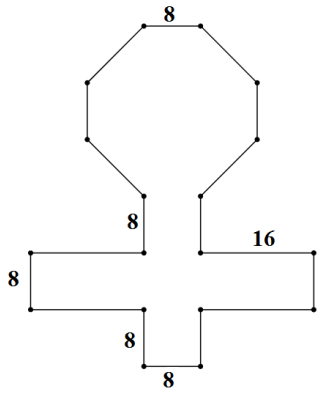
For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Anirudh, the tennis maestro, will one day fill the shoes of Rafael Nadal when he retires. He plays with an untraditional tennis racket in the shape of a triangle to get more spin. This triangle, labeled ABC, is isosceles with a vertex angle of 120^o. Given that $\overline{BC} = 10$ and $\overline{AB} = \overline{AC}$, what is the length of the altitude from point B to $\overline{AC}$?

   (A) $\frac{5\sqrt{3}}{3}$ (B) 10 (C) 5 (D) $5\sqrt{3}$ (E) NOTA

2. Sanjita loves to eat Sour Patch Kids. During one of her eating binges she came across an unusual one in the shape of a regular octagon on top of a vertical rectangle, with two identical rectangles attached to each side of the first rectangle. The diagram below contains the lengths of each respective side of the unusual Sour Patch Kid. What is the longest line segment that can be drawn between any two vertices that remains inside of the figure?



   (A) $8\sqrt{18+8\sqrt{2}}$ (B) $8\sqrt{26}$ (C) $20\sqrt{5}$ (D) $9\sqrt{30}$ (E) NOTA

3. Maxance enjoys playing soccer and he wants to buy a special edition Manchester City soccer ball from the mall. Assuming that the soccer ball is a polyhedron, if he counted 32 faces and 60 vertices on the ball, how many edges does the soccer ball have?

   (A) 30 (B) 90 (C) 28 (D) 92 (E) NOTA

4. Consider a triangle ABC with side lengths $AB = 18$, $BC = 18$, and $AC = 24$. What is the area of the circumcircle of this triangle?

   (A) 729π (B) $27\sqrt{5}\pi$ (C) 3645π (D) $\frac{729}{5}\pi$ (E) NOTA

5. Eccentric Siddharth wants to try octopus ice cream. He eats it in a cone that is composed of an upright frustum directly on top of an inverted frustum. The two frustums are adjoined at their larger bases. If the non-adjoined bases are closed off, the smaller radius of each frustum is 6, the larger radius is 16, and the height of the whole ice cream cone is 48, what is the total surface area of the ice cream cone?

   (A) 1216π (B) $(44\sqrt{601}+72)\pi$ (C) 572π (D) 1144π (E) NOTA

6. Consider a convex quadrilateral $ABCD$ with diagonals AC and BD that are perpendicular to one another. Given that the quadrilateral has side lengths $AB = 21$, $BC = 26$, and $CD = 32$, what is the length of side AD?

   (A) 27 (B) $\frac{336}{13}$ (C) 15 (D) 78 (E) NOTA

7. Samay is determined to figure out the shape of Anirudh's head and ends up sketching trapezoid $ABCD$. To make his diagram more elaborate, Samay also decides to draw Anirudh's hairline, which is represented by $\overline{HR}$. $\overline{AB}$ and $\overline{CD}$ are the bases of the trapezoid, and $\overline{AB} \parallel \overline{CD} \parallel \overline{HR}$. Point H lies along $\overline{AD}$ and point R lies along $\overline{BC}$. Given that $\overline{AB} = 12$, $\overline{CD} = 18$, $\overline{HR} = 17$, and $\overline{AD} = 15$, what is the length of $\overline{HD}$?

(A) 1 (B) $\frac{35}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) NOTA

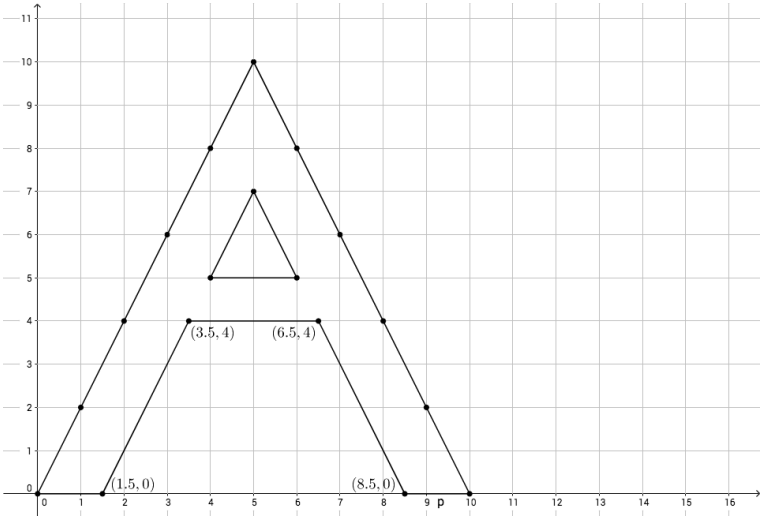
8. A circle with center O has a radius of 5 while a circle with center A has a radius of 13. Given that $\overline{OA} = 26$, what is the sum of the lengths of one common external and one common internal tangent between the two circles?

(A) $4\sqrt{22} + 6\sqrt{17}$ (B) $5\sqrt{19} + 3\sqrt{29}$ (C) $\sqrt{740} + 10\sqrt{10}$ (D) $\sqrt{740} + \sqrt{757}$ (E) NOTA

9. Jason Gao is known for having a massive diet and one essential food that he cannot live without is the doughnut. Assuming that the doughnuts Jason eats are in the exact shape of a torus with a major radius of 14 and a minor radius of 6, what is the volume of the doughnut?

(A) 1008π (B) 160π (C) 840π (D) $\frac{10112}{3}\pi$ (E) NOTA

10. Aarushi is obsessed with the show Pretty Little Liars and has spent much of her free time trying to figure out who "A" is. Every time Aarushi even sees the letter "A" anywhere, she becomes paranoid and believes that it's a sign telling her something. One day she finds a sticker depicting the letter "A" and wants to find its area. Using the diagram below drawn on a Cartesian plane, what value should Aarushi come up with?



(A) 15 (B) 30 (C) 28 (D) 35 (E) NOTA

11. Triangle ABC is inscribed in a circle with center X, where $\angle ABC = 90^o$, $\overline{BC} = 18$, and $\overline{AB} = 80$. The altitude from point B intersects $\overline{AC}$ at point D. What is the distance between point X and point D?

(A) $\frac{82}{3}$ (B) $\frac{1519}{41}$ (C) $\frac{1296}{101}$ (D) $\frac{1681}{40}$ (E) NOTA

12. In triangle ABC, $\overline{AB} = 13$, $\overline{BC} = 14$, and $\overline{AC} = 15$. What is the distance between the incenter and circumcenter of this triangle?

(A) $\frac{7\sqrt{5}}{12}$ (B) $\frac{3\sqrt{7}}{5}$ (C) $\frac{3\sqrt{5}}{2}$ (D) $\frac{\sqrt{65}}{8}$ (E) NOTA

13. In triangle ABC, $\overline{AB} = 7$, $\overline{BC} = 8$, and $\angle ABC = 60^o$. What is the length of $\overline{AC}$?

(A) $\sqrt{57}$ (B) $\sqrt{15}$ (C) $3\sqrt{6}$ (D) $2\sqrt{15}$ (E) NOTA

14. Alex and Rohan both love to run and decide to have a race. Alex runs along the line $y = -\frac{1}{3}x + 7$ and Rohan runs along the line $y = -\frac{1}{3}x - 3$. What is the shortest distance between the two lines that Alex and Rohan run along?

(A) 10 (B) 4 (C) $3\sqrt{10}$ (D) $3\sqrt{5}$ (E) NOTA

15. Out of the following choices, choose the undefined terms in geometry:
I. Ray
II. Plane
III. Line Segment
IV. Line
V. Angle

(A) I and II (B) II, IV, and V (C) I, II, IV, and V (D) II and IV (E) NOTA

16. Which of the following polygons is constructable with only a straightedge and compass?

(A) 32767-gon (B) 32769-gon (C) 65535-gon (D) 65537-gon (E) NOTA

17. Sri is a very bad player at any card game, so he decides that it is time to play a new game called liar's dice. The dice used are in the shape of a regular dodecahedron with side length 10. Sri wants to calculate the total surface area of this object. What value should Sri come up with for its total surface area? Express your answer in simplest radical form. (Hint: $\tan(36^o) = \sqrt{5 - 2\sqrt{5}}$)

(A) $(300\sqrt{5} + 750)(\sqrt{5 - 2\sqrt{5}})$
(B) $(600\sqrt{5} + 1500)((\sqrt{5 - 2\sqrt{5}})$
(C) $(120\sqrt{5} + 300)(\sqrt{5 - 2\sqrt{5}})$
(D) $(60\sqrt{5} + 150)(\sqrt{5 - 2\sqrt{5}})$
(E) NOTA

18. After losing to his brother in a swimming race, Isaiah gave up on the sport and is now afraid of going near a pool. If during a class field trip to a water park, Isaiah is at the point $(3, 4)$ and one of the pools is represented by the equation $x^2 + y^2 - 42x - 20y = -505$, what is the shortest distance between Isaiah and any point on the pool?

(A) 6 (B) $6\sqrt{10} - 6$ (C) $\sqrt{145}$ (D) $36 - 6\sqrt{10}$ (E) NOTA

19. A triangle has side lengths of 23, 42, and 65. Which of the following terms appropriately describes this figure?

(A) Obtuse (B) Equilateral (C) Orthogonal (D) Acute (E) NOTA

20. Ever since Fetty Wap released his debut album, Tanusri has been obsessed with the numbers 17 and 38. How many distinct obtuse triangles can Tanusri make with integer side lengths using both of these numbers as two of the triangle's side lengths?

(A) 23 (B) 24 (C) 25 (D) 26 (E) NOTA

21. Jason Zhang truly is a young romantic and at home he secretly puts his name with others on a love calculator. Given that the statement "If Jason gets a 100% chance with another person, then he will be happy" is true, which of the following statements must also be true?

(A) If he is happy, then Jason got a 100% chance with another person.
(B) If Jason doesn't get a 100% chance with another person, then he will not be happy.
(C) If he isn't happy, then Jason didn't get a 100% chance with another person.
(D) Jason is never happy.
(E) NOTA

22. Given a circle of radius 1, with an inscribed regular hexagon and an inscribed regular octagon, find the positive difference in the areas of the two shapes.

(A) $\sqrt{3}-\sqrt{2}$ (B) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (C) $\frac{4\sqrt{2}-3\sqrt{3}}{2}$ (D) $\frac{3\sqrt{3}}{2}-2\sqrt{2}$ (E) NOTA

23. There exists a regular dodecagon-shaped pen $ABCDEFGHIJKL$ with a side length of $10m$, and Bill Li the goat is tied to the outside of the pen at vertex A with a $50m$ rope. One day however, a terrible storm came through and broke open side EF of the pen, allowing Bill Li to enter the pen through only that opening. If Bill Li can now travel both inside and outside the pen, what is the total area of land, in m^2, that Bill Li can travel after the storm hits?

(A) 750π (B) $\frac{4375}{3}\pi$ (C) $\frac{6760}{3}\pi$ (D) 2000π (E) NOTA

24. Jeffrey Lu is bad at replying to private messages, and refuses to talk to anyone with an "o" in their name. Likewise, he is also afraid of circles in geometry and one problem in particular that haunted him read "What is the diameter of the circle with inscribed quadrilateral $ABCD$, given that $AC \perp BD$, $AB = 28$, $BC = CD$, and $[ABCD] = 1260$?" What is the correct answer to this question?

(A) 90 (B) 45 (C) 56 (D) 106 (E) NOTA

25. Josh's dream job is to become a plastic surgeon, and since Nihar happens to not have the sharpest of noses, he hopes to one day transform Nihar's nose into a regular tetrahedron. Given that the height of the nose Josh wants to form on Nihar is $2\sqrt{3}$, what would the volume of the nose be?

(A) 27 (B) 6 (C) 18 (D) 24 (E) NOTA

26. The Deerlake Math Team was arranged evenly in a circle. Bryan was standing in the 20th position and was directly opposite from Brighten who was standing in the 61st position. If each consecutive person standing on the circle had a line segment drawn between them forming a regular polygon, what would be the sum of the interior angles of this polygon?

(A) $14,220^o$ (B) $14,400^o$ (C) $14,580^o$ (D) $14,760^o$ (E) NOTA

27. Varun has a very "youthful" imagination, and believes that if he makes a paper airplane he will be able to fly on it across the world. He uses a 12 by 16 piece of rectangular paper, and folds it along its diagonal. What is the area of the region of overlap (the region where paper is on top of paper)?

(A) 117 (B) 128 (C) 75 (D) 96 (E) NOTA

28. Anagha and Tanvi both love to dance and decide to go on a shopping spree around town to find the perfect outfit. Anagha lives at the point (0,0) and Tanvi lives at the point (3,4). Anagha drives to Tanvi's house first to pick her up, and then they go to Oscar de la Renta at point (14,7), Louis Vuitton at point (24,10), and finally to Prada at point (26,17) in that respective order. Given that they can only travel up or to the right one unit at a time, what are the last two digits of the total number of possible paths for the entire trip? Consider this problem on a standard Cartesian plane.

(A) 42 (B) 30 (C) 60 (D) 40 (E) NOTA

29. Rayyan wishes to expand his collection of watches. He walks into a Tag Heuer shop and on the first watch he notices that the smaller angle between the hour and minute hand is 41^o. Given that noon happened less than an hour ago, what time does the watch read?

(A) 12:59 PM (B) 12:58 PM (C) 12:48 PM (D) 12:52 PM (E) NOTA

30. Rayyan dreams that one day he will become the starting quarterback for the New England Patriots. In order to do so, the Patriots have a set ratio of the volume of one's biceps to their height that must be met. The current quarterback, Tom Brady, is 76 in. tall and has biceps in the shape of a cylinder with radius of 4 in. and a length (height) of 7 in. Given that Rayyan is 68 in. tall and has biceps in the shape of a sphere, what must the radius of his biceps be, in inches, so that he meets the requirements?

(A) $\sqrt[3]{\frac{1428}{19}}$ (B) $\sqrt[3]{\frac{7616}{57}}$ (C) $\sqrt[3]{\frac{2499}{19}}$ (D) $\sqrt[3]{\frac{13,328}{57}}$ (E) NOTA